Combinatorics and Graph Theory

M. Math. II

Back Paper Examination

Instructions: All questions carry ten marks. All graphs are assumed to be simple.

1. Prove that in any non-trivial Steiner system $S(t, k, v)$, we must have
$$v \geq (t+1)(k-t+1)$$
2. State and prove Fisher's inequality.
3. Let k be a natural number and let Q_k denote the graph whose vertices are k-tules with entries in $\{0, 1\}$ and edges are pairs of k-tuples that differ in exactly one position. Prove that the complete bipartite graph $K_{2,3}$ is not a subgraph of Q_k.
4. Prove or disprove: Every graph contains at least one non cut-vertex.
5. Define a tree. Let m be a natural number and suppose that a tree T has exactly one vertex of degree i for $2 \leq i \leq m$ and all other vertices of degree 1. Compute the number of vertices of T.
6. Define an Eulerian graph. State and prove a necessary and sufficient condition for a connected graph to be Eulerian.
7. Define Hamiltonian graph. Prove that given a subset S of vertices of a Hamiltonian graph G, the induced graph on $V(G) \setminus S$ must have at most at most $\mid S \mid$ connected components. Decide, with justification, whether the converse of this statement true or not.
8. Define a matching. Prove that any k regular bipartite graph has a perfect matching.
9. Let G be a k-regular graph with $k > 1$. Prove that G has k as an eigen value and compute its multiplicity.
10. Define planar graph. If G is a planar graph on n vertices with girth at least 4, then prove that it can have at most $2n - 4$ edges.